PHILOSOPHICAL TRANSACTIONS.

I. *On the Evidence of the existence of the Decennial Inequality in the Solar-diurnal Magnetic Variations, and its non-existence in the Lunar-diurnal Variation, of the Declination at Hobarton. By Major-General* EDWARD SABINE, *of the Royal Artillery, Treas. and V.P. of the Royal Society.*

IN a report presented to the British Association at Liverpool in September 1854, entitled " On some of the results obtained at the British Colonial Magnetic Observatories," I stated that, as far as my examination of the observations had then gone, I had found in the Lunar-diurnal magnetic variation no trace of the *decennial period* which is so distinctly marked in all the variations connected with the Sun. And in a subsequent communication to the Royal Society in June 1856, " On the Lunar-diurnal Variation at Toronto," in which the moon's influence on each of the three magnetic elements was examined, the conclusion arrived at was to the same effect, viz. that the observations at Toronto " showed no appearance of the decennial period which constitutes so marked a feature in the solar-diurnal variations."

Since these statements were made, I have read M. KREIL's memoir " On the Influence of the Moon on the horizontal component of the Magnetic Force," presented to the Imperial Academy of Sciences at Vienna in 1852 and printed in 1853, from which I learn (pp. 45, 46) that M. KREIL is of opinion that the observations of different years at Milan and Prague, when combined, would rather favour the supposition that the same decennial period which exists in the solar variation affects also the lunar magnetic influence. The question is one of such manifest importance in its theoretical bearing, that I considered it desirable to lose no time in re-examining it by the aid of the Hobarton observations, which, as it appeared to me, were particularly suitable for the purpose, inasmuch as they consist of eight consecutive years of hourly observation (from January 1841 to December 1848 inclusive), made with one and the same set of instruments, and by a uniform system of observation. The results of this examination have been, as it appears to me, so decidedly confirmatory of the conclusions drawn from the Toronto observations, both as regards the existence of the decennial period in the

B

two classes of solar-diurnal variation (viz. in the mean diurnal variation occasioned by the disturbances of large amount, and in what may be termed the more regular solar-diurnal variation), and the non-existence of a similar decennial period in the case of the lunar-diurnal variation, that I have been induced to make these results the subject of a communication to the Royal Society.

The observations at Hobarton comprise, so far as I am aware, the most extensive consecutive series of magnetic observations that have hitherto been made. The *hourly* system was adopted there from the first commencement of the Observatory. We owe it to the zeal and clear-sightedness of Captain (now Admiral) Sir JAMES CLARK ROSS, by whom the Hobarton Observatory was established, that this improvement upon the two-hourly system proposed by the Royal Society was introduced. Its superiority for many of the contemplated objects of the Observatories was speedily recognized, and the example followed at the Ordnance Colonial Observatories, by the employment of one additional non-commissioned officer at each, thus enabling the number of daily observations to be doubled at the small extra expense of about £24 a-year for each observatory. The duties of "Observer," which at the Ordnance establishments were performed by non-commissioned officers of the Royal Artillery, devolved at Hobarton upon the officers of the Royal Navy who formed the *personnel* of that Observatory: and I gladly avail myself of this opportunity of placing on record, that to the steady and unremitting zeal of Commander JOSEPH HENRY KAY during the whole period of eight years, of Lieutenants PETER SCOTT and JOSEPH DAYMAN for the first four years, and of Lieutenants ALEXANDER SMITH and FRANCIS SIMPKINSON during the last four years, are primarily due whatever results may now or hereafter be derived from the Hobarton observations *.

The observations of each month, arranged in the same forms as those of the Ordnance establishments, were transmitted by the Admiralty to my office at Woolwich, and have been printed. For the purpose of this examination they have been treated precisely as those of Toronto, as described in the Philosophical Transactions for 1856, Art. XVI. and Art. XXII. The aggregate number of the observations of the Declination at Hobarton which have been passed through those processes of reduction is 56,202; of these, 4207 were found to differ from the monthly normal at the same hour by an amount which equalled or exceeded four scale-divisions, or 2'·84 in arc value, and were accordingly separated from the rest, for the purpose of deducing from them the periodical laws of the class of phenomena to which they belong. The monthly normals, derived from the great body of the observations after the exclusion of the disturbances of four scale-divisions and upwards, have been employed to show the variation in amount in the different years of the mean diurnal solar influence, forming what is generally understood as the regular solar-diurnal variation; and the same observations, 51,995 in number, rearranged according to the lunar hours to which they respectively most nearly approximate, have supplied the mean lunar-diurnal variation in the different years treated of in this communication.

* In mentioning the names of the observers, that of Mr. JEFFREY, for several years assistant to Captain KAY, should not be omitted.

Disturbance-variation.—Before, however, we enter on the lunar question, I wish to show the accordance of the two classes of the solar-diurnal phenomena at Hobarton with the decennial period, as it has been inferred from observations in other parts of the globe; and I shall commence with the *disturbance-variation*, because it was from this branch of the phenomena that the decennial connexion between the solar-magnetic variations and those of the solar spots was first inferred; and because, on account of the labour which is required in the investigation, the periodical laws of the disturbances have hitherto been made out at no other observatories than those of the British Colonies. I gladly avail myself therefore of the opportunity of showing, that when the disturbances, occurring at Hobarton during eight years, are broken into four distinct and equal portions, each of two years' duration, each such portion manifests, under a suitable mode of investigation, the same periodical law of diurnal variation, almost identical in the principal features of direction and turning hours, and differing only in the magnitude of the variation in different years, in which difference it conforms strictly to the decennial period as indicated elsewhere, having a minimum in 1843—1844, and a maximum five years later.

As the existence of a law of diurnal variation regulating the occurrence of the disturbances of large amount rests hitherto, I believe, only on investigations made by myself, I may be excused for here calling attention to the fact, that each of the four portions, into which the disturbances at Hobarton have been divided, bears its testimony to the existence of this law;—which may be accounted a general one, since it has been found to prevail at stations so widely distant from each other as Toronto, St. Helena, and Hobarton.

The following Table exhibits, for each of the two-yearly periods, the excess at the several hours of mean solar time at the station, of the aggregate easterly differences from the monthly normal at the same hour over the aggregate westerly differences (or westerly over easterly as the case may be), divided by the number of days of observation in each two-yearly period: it is therefore strictly a table of the mean diurnal variation due to the disturbances that have been separated for the purpose of investigating the periodical laws of this class of the phenomena.

TABLE I.

Hours.	1841 and 1842.	1843 and 1844.	1845 and 1846.	1847 and 1848.	Hours.	1841 and 1842.	1843 and 1844.	1845 and 1846.	1847 and 1848.
Noon.	0·00	+0·03	+0·10	+0·09	12	−0·53	−0·38	−0·45	−0·65
1	+0·10	+0·11	+0·08	+0·03	13	−0·30	−0·23	−0·39	−0·54
2	0·00	+0·11	+0·03	−0·02	14	−0·20	−0·14	−0·35	−0·36
3	+0·05	+0·12	+0·07	+0·20	15	−0·17	−0·05	−0·19	−0·33
4	+0·15	+0·06	+0·07	+0·23	16	+0·05	+0·03	−0·05	−0·28
5	+0·13	+0·03	+0·03	+0·32	17	+0·08	+0·09	+0·01	±0·03
6	−0·04	−0·05	−0·05	+0·20	18	+0·13	+0·09	+0·04	+0·10
7	−0·17	−0·17	−0·11	−0·10	19	+0·20	+0·09	+0·17	+0·22
8	−0·43	−0·30	−0·22	−0·26	20	+0·27	+0·11	+0·10	+0·23
9	−0·42	−0·43	−0·37	−0·39	21	+0·14	+0·06	+0·15	+0·10
10	−0·55	−0·36	−0·41	−0·64	22	+0·05	+0·06	+0·09	+0·19
11	−0·59	−0·43	−0·46	−0·82	23	+0·01	+0·05	+0·12	+0·13

The values are decimals of a minute of arc. The sign $+$ corresponds to Easterly and $-$ to Westerly disturbance.

These variations may be represented by the following formulæ:—

1841—1842 . . $\Delta_x = -0 \cdot 085 - 0 \cdot 299 \sin(a + 294 \cdot 7) + 0 \cdot 178 \sin(2a + 298 \cdot 4)$

1843—1844 . . $\Delta_x = -0 \cdot 062 - 0 \cdot 231 \sin(a + 294 \cdot 1) + 0 \cdot 126 \sin(2a + 319 \cdot 7)$

1845—1846 . . $\Delta_x = -0 \cdot 083 - 0 \cdot 273 \sin(a + 283 \cdot 9) + 0 \cdot 107 \sin(2a + 280 \cdot 2)$

1847—1848 . . $\Delta_x = -0 \cdot 097 - 0 \cdot 447 \sin(a + 300 \cdot 4) + 0 \cdot 227 \sin(2a + 278 \cdot 6)$

Δ being the variation at x, the hour required, and a (corresponding to x) counted in hours from noon and multiplied by 15°.

The curves computed by these formulæ are shown to the eye in seconds of arc in Plate I. fig. 1. It will at once be seen how thoroughly their inflections accord in character, and at the same time how decidedly and systematically they differ in amount. The inflections are least in the curve derived from the years 1843—1844, and greatest in that derived from the years 1847—1848. The hour of greatest westerly deflection is about 11 P.M., and that of greatest easterly deflection about 7 A.M. The difference between the extreme deflections at these hours, or the range of the variation in twenty-four hours in seconds of arc, is given by the formulæ as follows:—

	1841—1842.	1843—1844.	1845—1846.	1847—1848.
Easterly extreme	12·0	6·8	8·8	23·7
Westerly extreme	33·5	24·7	27·5	44·2
Sum	45·5	31·5	36·3	67·9

Taking the differences between the actual results of the observations at the same hours, or the range of the variation in twenty-four hours, from direct observation, we have as follows:—

	1841—1842.	1843—1844.	1845—1846.	1847—1848.
Easterly extreme	12·0	5·4	11·2	13·2
Westerly extreme	35·4	25·8	27·6	49·2
Sum	47·4	31·2	38·8	62·4

Thus it will be seen, that if we estimate the variation in the mean effect of the disturbances, in the course of the decennial period, by the difference in their mean diurnal range respectively in 1843—1844 and 1847—1848, we should infer that their mean effect is at least twice as great in the year of maximum as in the year of minimum.

It is probable that if either a higher or a lower arbitrary standard than four scale-divisions were taken as constituting a "large disturbance," the inequality between the years of maximum and minimum might be rendered somewhat greater or somewhat less, but still not materially so; the result is of too decided a character to doubt the evidence it affords of a subsisting substantial difference between the years of maximum and minimum.

Direct Solar-diurnal Variation.—Table II. exhibits the mean solar-diurnal variation for the same four periods, each of two years' duration, derived from the observations after the exclusion of the larger disturbances. The sign + signifies Easterly and — Westerly differences from the mean or normal direction in the twenty-four hours.

TABLE II.

Hours.	1841 and 1842.	1843 and 1844.	1845 and 1846.	1847 and 1848.	Hours.	1841 and 1842.	1843 and 1844.	1845 and 1846.	1847 and 1848.
Noon.	+1·61	+1·29	+1·20	+1·00	12	−1·01	−0·87	−1·04	−0·89
1	+3·46	+3·19	+3·42	+3·79	13	−0·77	−0·68	−0·86	−0·65
2	+4·42	+4·03	+4·54	+5·25	14	−0·56	−0·41	−0·62	−0·49
3	+4·10	+3·85	+4·46	+5·01	15	−0·33	−0·26	−0·42	−0·36
4	+3·04	+2·96	+3·44	+3·83	16	−0·41	−0·33	−0·46	−0·43
5	+1·82	+1·82	+2·08	+2·23	17	−0·76	−0·59	−0·86	−0·95
6	+0·99	+0·98	+1·11	+1·37	18	−1·30	−1·07	−1·26	−1·40
7	+0·36	+0·40	+0·46	+0·80	19	−2·13	−1·85	−1·83	−2·53
8	−0·16	−0·11	−0·07	+0·27	20	−3·04	−2·88	−3·00	−3·74
9	−0·72	−0·55	−0·52	−0·25	21	−3·36	−3·29	−3·61	−4·51
10	−1·03	−0·92	−0·91	−0·64	22	−2·46	−2·71	−2·92	−3·92
11	−1·11	−0·95	−1·09	−0·90	23	−0·63	−0·98	−1·19	−1·91

In all the four periods the greatest easterly deflection occurs at 2 P.M., and the greatest westerly at 9 A.M. The extreme differences are,—

	1841—1842.	1843—1844.	1845—1846.	1847—1848.
2 P.M. Easterly	4·42	4·03	4·54	5·25
9 A.M. Westerly	3·36	3·29	3·61	4·51
Sums . . .	7·78	7·32	8·15	9·76

Here also, if we estimate the decennial variation by the difference in the range of the diurnal movement, we find 1843—1844 the years of minimum and 1847—1848 those of maximum; although the inequality between the extremes is not so great as in the disturbance-variation.

We obtain this result from the eight years in which the observations were made hourly; but we have also observations in the six following years made with the same instruments at the hours of 2 P.M. and 6 A.M., and we are thus enabled to trace the decennial inequality through a period of fourteen consecutive years. In so doing we have to employ throughout (*i. e.* in the eight years as in the six years) the mean positions at 2 P.M. and 6 A.M. only. The range in the different years is here strictly comparative, but it is not of course the extreme range. Table III. shows these particulars.

TABLE III.   Hobarton Declination.

Years.	Mean scale reading at		Differences.—2 P.M.—6 A.M.	
	2 P.M.	6 A.M.	Scale-divisions.	Arc.
1841	61·5	53·6	7·9	5·61
1842	83·3	75·9	7·4	5·25
1843	83·8	76·4	7·4	5·25
1844	81·6	74·4	7·2	5·11
1845	85·0	76·5	8·5	6·03
1846	87·1	79·2	7·9	5·61
1847	90·0	79·7	10·3	7·31
1848	91·5	80·3	11·2	7·95
1849	91·8	81·6	10·2	7·24
1850	91·1	82·3	8·8	6·25
1851	90·7	81·9	8·8	6·25
1852	90·3	80·7	9·6	6·82
1853	81·5	74·0	7·5	5·32
1854	76·8	68·4	8·4	5·96

This Table affords a striking illustration of the systematic character of the decennial inequality; and with the evidence previously adduced in reference to the mean diurnal effect of the greater magnetic disturbances in different years, shows that in both classes of solar-diurnal variation, (the daily and constant, and that due to disturbances of occasional occurrence,) the decennial inequality is as distinctly and decidedly marked in the Hobarton observations as it has been found to be elsewhere.

Lunar-diurnal Variation.—Table IV. exhibits the lunar-diurnal variation in the four periods, each of two years' duration, obtained from the 51,995 observations remaining after the separation of the disturbances of large amount, rearranged according to lunar hours and treated in the manner described in the Philosophical Transactions for 1856, Art. XXII.   The sign + signifies Easterly, and the sign — Westerly deflections from the normal direction.

TABLE IV.

One scale-division $= 0'·71$.

Lunar hours.	1841 and 1842.	1843 and 1844.	1845 and 1846.	1847 and 1848.	Lunar hours.	1841 and 1842.	1843 and 1844.	1845 and 1846.	1847 and 1848.
	Sc. div.	Sc. div.	Sc. div.	Sc. div.		Sc. div.	Sc. div.	Sc. div.	Sc. div.
0	+ 0·14	+ 0·18	+ 0·12	+ 0·27	12	+ 0·14	+ 0·15	+ 0·16	+ 0·02
1	+ 0·08	+ 0·24	+ 0·14	+ 0·24	13	+ 0·26	+ 0·27	+ 0·25	+ 0·15
2	+ 0·07	+ 0·16	+ 0·25	+ 0·23	14	+ 0·19	+ 0·28	+ 0·25	+ 0·09
3	− 0·06	+ 0·21	+ 0·28	+ 0·10	15	+ 0·13	+ 0·22	+ 0·14	− 0·02
4	− 0·12	+ 0·13	+ 0·26	+ 0·08	16	+ 0·09	+ 0·14	+ 0·05	− 0·01
5	− 0·14	− 0·09	+ 0·11	− 0·09	17	+ 0·08	− 0·08	− 0·09	− 0·10
6	− 0·27	− 0·14	− 0·04	− 0·13	18	− 0·14	− 0·17	− 0·19	− 0·15
7	− 0·20	− 0·22	− 0·04	− 0·18	19	− 0·13	− 0·25	− 0·30	− 0·26
8	− 0·16	− 0·25	− 0·13	− 0·06	20	− 0·12	− 0·27	− 0·40	− 0·15
9	− 0·06	− 0·09	− 0·09	− 0·06	21	− 0·03	− 0·23	− 0·30	− 0·15
10	− 0·04	− 0·03	− 0·06	+ 0·01	22	+ 0·02	− 0·01	− 0·24	+ 0·09
11	− 0·01	+ 0·11	+ 0·07	+ 0·08	23	+ 0·08	+ 0·03	− 0·05	+ 0·18

The lunar-diurnal variation for these periods may therefore be respectively represented by the following formulæ, in which the coëfficients are expressed in seconds of arc, and a is reckoned in hours (multiplied by 15°) from the epoch of the moon's superior culmination.

1841—1842 . . . $\Delta_x = -0\overset{''}{\cdot}35 - 3\overset{''}{\cdot}61 \sin(a + 21\overset{\circ}{\cdot}8) + 6\overset{''}{\cdot}69 \sin(2a + 61\overset{'}{\cdot}8)$.

1843—1844 . . . $\Delta_x = +0\cdot51 + 1\cdot08 \sin(a + 284\cdot4) + 10\cdot7 \sin(2a + 43\cdot6)$.

1845—1846 . . . $\Delta_x = +0\cdot26 + 5\cdot14 \sin(a + 339\cdot2) + 10\cdot4 \sin(2a + 27\cdot6)$.

1847—1848 . . . $\Delta_x = +0\cdot32 + 2\cdot7 \sin(a + 55\cdot1) + 7\cdot7 \sin(2a + 65\cdot0)$.

Mean of the eight years $\Delta_x = +0\cdot25 + 0\cdot98 \sin(a + 318\cdot4) + 8\cdot75 \sin(2a + 45\cdot8)$.

We may at once perceive, from the relative magnitude of the coefficient of principal significance (that of sin $2a$), that the differences in the lunar-diurnal variation corresponding to different years show no conformity to the inequality manifested in those of the solar-diurnal variations.

The curves severally corresponding to these formulæ are shown in Plate I. fig. 2. for each of the four two-yearly periods. They exhibit such differences as are to be expected from periods of such limited duration; but the differences present no appearance whatsoever of a systematic variation corresponding to the decennial inequality of the solar-diurnal variations. There are in each curve four extremes nearly equidistant from each other, two of which are easterly and two westerly extremes: if in each period we take the sum of the extremes, not indeed as an exact measure, but as furnishing an approximate indication of the relative magnitudes of the respective variations, the periods arrange themselves in the following order:—

1. 1843—1844. Sum of extremes . . . . . . $41\overset{''}{\cdot}8$
2. 1845—1846. Sum of extremes . . . . . . $41\cdot6$
3. 1847—1848. Sum of extremes . . . . . . $30\cdot6$
4. 1841—1842. Sum of extremes . . . . . . $28\cdot1$

If we should prefer as a criterion of the magnitude of the variation, the sum of all the deflections at the different hours in the respective periods, we should have the succession as follows, taking the deflections as calculated by the formulæ:—

1. 1845—1846. Sum of deflections . . . . $169\overset{''}{\cdot}9$
2. 1843—1844. Sum of deflections . . . . $165\cdot5$
3. 1847—1848. Sum of deflections . . . . $120\cdot5$
4. 1841—1842. Sum of deflections . . . . $113\cdot4$

Or taking the deflections as actually observed at the different hours:—

1. 1845—1846. Sum of deflections . . . . $170\overset{''}{\cdot}8$
2. 1843—1844. Sum of deflections . . . . $168\cdot3$
3. 1847—1848. Sum of deflections . . . . $123\cdot5$
4. 1841—1842. Sum of deflections . . . . $117\cdot6$

In both classes of *solar* variations the years 1841, 1846, 1847 and 1848 are the years of greatest range, and the years 1842, 1843, 1844 and 1845 those of least range. If we unite in one curve the *lunar* results in the four first-named years, and in a second curve the results in the four last-named years, and compare these two curves, we are so far from finding in the lunar variation an inequality corresponding to that in the solar variation, that the difference between the curves, such as it is, is in fact in the opposite direction. These curves are drawn in Plate I. fig. 3, and are there shown in comparison with a third curve, which is that of the mean lunar-diurnal variation in the entire period of eight years.

The Tables I. to IV. were prepared, under Mr. MAGRATH's superintendence, by the Non-Commissioned Officers of the Royal Artillery employed in my office, each entry having been verified by two independent computers. The observations themselves are printed in the first three volumes of the 'Hobarton Observations.'

Hobarton — Magnetic Declination.

Solar Astronomical Hours.

Scale. | East | West | Scale.

Mean or normal Line of the Declination

Fig. 1.

Mean diurnal Variation

caused by the

Large Magnetic Disturbances.

Solar Astronomical Hours.

Lunar Hours.

Scale. | Scale.

Fig. 2. Mean Lunar-diurnal Variation.

East | West | East | West

Mean or normal Line of the Declination.

Fig. 3. Mean Lunar-diurnal Variation.

East | West | East | West

Mean or normal Line of the Declination

Lunar Hours